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# LUNAR PHYSICAL PARAMETERS STUDY

## PARTIAL REPORT NO. 8

## DESIGN CALCULATIONS

## MEASUREMENT OF THERMAL DIFFUSIVITY

May 8, 1961

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LUNAR PHYSICAL PARAMETERS STUDY

DESIGN CALCULATIONS

MEASUREMENT OF THERMAL DIFFUSIVITY

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# MEASUREMENT OF THERMAL DIFFUSIVITY

Equipment is to be assembled to test the feasibility of the methods proposed to measure the thermal diffusivity of the lunar surface material. This measurement is to be made directly on the surface and also within a drilled hole. This present report outlines the design calculations for this instrumentation, reviewing the several optional designs.

## Surface Measurement

At surface, the thermal diffusivity is to be deduced from the change in temperature experienced within a region of known geometry as a result of abruptly altering the incoming radiation within this region. This change in the incoming radiation can be the result of either shining an infrared lamp on the desired area, or abruptly shielding the surface from direct solar radiation. It is necessary to estimate the shield dimensions or, optionally, the lamp dimensions and power, to give measurable temperature changes for a diffusivity measurement. At this point it is not necessary to solve exactly the heat flow equations for the specific situation existing during the diffusivity measurement on the lunar surface. Rather, one can select a system which approximates the lunar conditions and obtain, rather easily, solutions of sufficient accuracy for engineering design. This treatment for the surface diffusivity determination is given in Appendix A. The conclusions reached in this treatment are as follows:

1. Measurement of thermal diffusivity can be accomplished on surface by the use of a shield system having a radius of

about 11 cm., or by use of a lamp system irradiating an area of surface having a radius of at least 11 cm.

2. The depth of surface affected by this treatment is on the order of 4 cm. in the time periods required for measurement.
3. With radiometer sensitivities available it is not practical to attempt a measurement to separate thermal conductivity from thermal diffusivity.

Downhole Measurement

The basis of study of downhole equipment is given in Appendix B, but the work has not progressed to the point allowing conclusions or data to be reported.

## APPENDIX A

## DESIGN CALCULATIONS FOR SURFACE THERMAL DIFFUSIVITY DETERMINATIONS

For the design of the surface thermal diffusivity apparatus, the system to be considered is a "semi-infinite" region $z > 0$ initially at zero temperature throughout $z > 0$. Carslaw and Jaeger[1] treats these systems for various heating geometries at the surface $z = 0$. The Carslaw and Jaeger solution for a supply of heat at the rate of q calories per unit time per unit area for $t > 0$ over the circle $x^2 + y^2 < a^2$, $z = 0$ evaluated at the point $(0, 0, z)$ is

$$\theta = \frac{2q(\alpha t)^{1/2}}{K} \left\{ \operatorname{ierfc} \frac{z}{2(\alpha t)^{1/2}} - \operatorname{ierfc} \frac{(z^2 + a^2)^{1/2}}{2(\alpha t)^{1/2}} \right\} . \qquad (1)$$

Note that $\operatorname{ierfc} \xi = \frac{1}{\pi^{1/2}} e^{-\xi^2} - \xi \operatorname{erfc} \xi$, and $\operatorname{erfc} \xi = 1 - \operatorname{erf} \xi$, so that

$$\theta = \frac{2q(\alpha t)^{1/2}}{K} \left[ \frac{1}{\pi^{1/2}} e^{\frac{-z^2}{4\alpha t}} - \frac{z}{2(\alpha t)^{1/2}} \operatorname{erfc} \frac{z}{2(\alpha t)^{1/2}} - \frac{1}{\pi^{1/2}} e^{\frac{-z^2 + a^2}{4\alpha t}} + \frac{(z^2 + a^2)^{1/2}}{2(\alpha t)^{1/2}} \operatorname{erfc} \frac{(z^2 + a^2)^{1/2}}{2(\alpha t)^{1/2}} \right] . \qquad (2)$$

Evaluated at $z = 0$, equation (2) becomes,

[1] Conduction of Heat in Solids, H. S. Carslaw and J. C. Jaeger, Second Edition, Oxford at the Clarendon Press, 1959

$$\theta = \frac{2q(\alpha t)^{1/2}}{K} \left\{ \frac{1}{\pi^{1/2}} - \frac{e^{-\frac{a^2}{4\alpha t}}}{\pi^{1/2}} + \frac{a}{2(\alpha t)^{1/2}} \operatorname{erfc} \frac{a}{2(\alpha t)^{1/2}} \right\}, \quad (3)$$

$$\theta = \frac{2q(\alpha t)^{1/2}}{K\,\pi^{1/2}} \left(1 - e^{-\frac{a^2}{4\alpha t}}\right) + \frac{q\,a}{K} \operatorname{erfc} \frac{a}{2(\alpha t)^{1/2}} . \quad (4)$$

So far this has been straightforward manipulation of the Carslaw and Jaeger solution, and in this form it gives the temperature, $\theta$, at the center of a circle, radius a, as a function of time, t, for a medium (semi-infinite) $z > 0$ initially at zero temperature throughout, having a heat input over $x^2 + y^2 < a^2$ of q calories per $cm^2$ per second (no heat flow over the rest of the boundary $z = 0$), having thermal conductivity K and thermal diffusivity $\alpha = \frac{K}{\rho c}$ (where $\rho$ is density, c is specific heat).

This particular solution is now to be used in various ways to estimate necessary shield dimensions and manipulations, or, optionally, infrared lamp powers and manipulations, to accomplish the measurement of thermal diffusivity at the surface.

The first point considered is the radius of affected area, a, necessary to give measurable temperature differences in reasonable time periods. A radiometer viewing a total included angle of 22° from 1 ft. above surface is averaging temperatures over an area having a radius 30 tan 11° ≈ 5.8 cm. The specification for the affected area must say that over the area viewed by the radiometer the temperature change $\theta$ is constant in reasonable time periods. It seems obvious that as K goes to zero more abrupt changes in $\theta$ would appear at the boundary and a uniform temperature (in terms of the space variables) would appear over the area $x^2 + y^2 < a^2$. The dimension, a, will be deduced for the highest value of conductivity expected, anticipating that it will then be usable for all lower values. The specification for the highest value of thermal conductivity expected on the lunar surface is $10^{-4} \frac{\text{calories}}{\text{cm.sec.}^\circ\text{C}}$. If the surface is a material having density and specific heat approximating the values for sand, $\rho c \approx 0.4$. It is necessary to consider also the case of a low density material, so arbitrarily a $\rho c$ value $\approx 0.04$ is also to be taken. From the Carslaw and Jaeger solution (Eqn. 4) values of "a" will be determined at which, for ranges of $\alpha$ and t, $\theta$ is independent of a. (No claims will be made for the rigor of any of the following treatments.) It can be reasoned that $\theta$ is independent of a when $e^{-a^2/4\alpha t} << 1$, with the additional condition that

$$\frac{q\,a}{K} \operatorname{erfc} \frac{a}{2(\alpha t)^{1/2}} << \frac{2q(\alpha t)^{1/2}}{K\,\pi^{1/2}},$$

leaving an expression for $\theta$ in which "a" does not appear

$$\theta = \frac{2q(\alpha t)^{1/2}}{K\,\pi^{1/2}}. \tag{5}$$

It remains to establish the values of $\alpha$, a, and t for which these conditions hold. Taking the original expression, Eqn. 4, and rewriting,

$$\theta = \frac{2q(\alpha t)^{1/2}}{K\,\pi^{1/2}} \left\{1-e^{-a^2/4\alpha t} + \frac{a\,\pi^{1/2}}{2(\alpha t)^{1/2}} \operatorname{erfc} \frac{a}{2(\alpha t)^{1/2}}\right\}. \tag{6}$$

Considering only the function within { }, it takes the form,

$$1 - e^{-\xi^2} + \pi^{1/2}\,\xi \text{ erfc } \xi, \tag{7}$$

where

$$\xi = \frac{a}{2(\alpha t)^{1/2}} \cdot$$

Values of this function were computed for several values of $\xi$ and its behavior is shown in Fig. 1. For practical purposes this function becomes unity for values of $\xi > 2.5$. Therefore, for

$$\xi = \frac{a}{2(\alpha t)^{1/2}} > 2.5,$$

$$\theta = \frac{2q(\alpha t)^{1/2}}{K\pi^{1/2}}\text{, an expression independent of } a. \tag{8}$$

If $K = 10^{-4}\ \frac{\text{calories}}{\text{cm.sec.}^\circ\text{C}}$, $\rho c = 0.4$, then $\alpha = 2.5\text{x}10^{-4}$ $\text{cm}^2$/sec. If $K = 10^{-4}\ \frac{\text{calories}}{\text{cm.sec.}^\circ\text{C}}$, $\rho c = 0.04$, then $\alpha = 2.5\text{x}10^{-3}$ $\text{cm}^2$/sec. In later sections it will be demonstrated that for $\alpha = 2.5\text{x}10^{-4}$ $\text{cm}^2$/sec. a measurement of thermal diffusivity can be made in 4000 sec., and for $\alpha = 2.5\text{x}10^{-3}$ $\text{cm}^2$/sec., a measurement can be made in 400 sec. In each case the product $\alpha t = 1$. In order that $\frac{a}{2(\alpha t)^{1/2}} > 2.5$ for each of the above cases $a > 5$ cm. The temperature change at the center of a circle, radius = 5 cm. is independent of the radius if the medium has a thermal diffusivity less than or equal to $2.5\text{x}10^{-4}$ $\text{cm}^2$/sec. and the temperatures are measured for times less than 4000 sec. This is also correct within this radius of 5 cm. if the medium has a thermal diffusivity equal to $2.5\times10^{-3}$ $cm^2$/sec. and the temperatures are measured for times less than 400 sec. Earlier in the discussion the point was made that the radiometer for temperature measurement would be viewing a circle of radius about 6 cm. For this radius to enclose a region of temperature independent of the boundaries, the area in which incoming radiation is altered must be a circle of radius greater than 6 + 5 = 11 cm. Fig. 2 will aid in understanding this reasoning, remembering that the inner 6 cm. radius circle is viewed by the radiometer, and the 5 cm. circles illustrate the extent of area within which, for these time limits, the temperature has radius as a parameter. Every point on the circumference of the circle enclosing the radiometer view is a minimum of 5 cm. from the boundary of the area affected, which should give within the view of the radiometer temperatures unaffected by radius as a parameter.

Similar reasoning will allow an estimate of depth affected during this measurement. Going back to one of the original equations, Eqn. 2, evidence has been given that, at $z = 0$, for $\alpha t = 1$ all terms including 'a' effectively disappear for values of $a > 5$ cm. Going to the case for $\alpha t = 1$, at $z \neq 0$, it is obvious that all terms including 'a' remain negligible for values of $a > 5$ cm., leaving,

$$\theta = \frac{2q(\alpha t)^{1/2}}{K} \left[ \frac{e^{-z^2/4\alpha t}}{\pi^{1/2}} - \frac{z}{2(\alpha t)^{1/2}} \operatorname{erfc} \frac{z}{2(\alpha t)^{1/2}} \right]. \quad (9)$$

The expression within [ ] is the function $\frac{e^{-\xi^2}}{\pi^{1/2}} - \xi \operatorname{erfc} \xi$

as plotted in Fig. 1, which goes to a negligibly small value near $\xi = \frac{z}{2(\alpha t)^{1/2}} = 2$.

For the case $\alpha t \leq 1$, $z = 4$ cm. is the maximum depth affected by the alteration at surface, independent of the radius of affected area. It is now possible to estimate temperature histories at the point (0, 0, 0) which is the center of the area of altered radiation. Evidence has been given that, for $\alpha t \leq 1$, $a \geq 5$, Equation 4 reduces to

$$\theta = \frac{2q(\alpha t)^{1/2}}{K\,\pi^{1/2}}, \tag{5}$$

where $\theta$ is the temperature change noted at (0, 0, 0) at time, t, for an input of heat of q calories per cm$^2$ per sec. for a semi-infinite medium, $z > 0$, initially at uniform temperature throughout and having a value of thermal diffusivity $\alpha = \frac{K}{\rho c}$ such that $\alpha t \leq 1$. This reduced equation is the exact equivalent of the solution as given by Carslaw and Jaeger for a constant flow of heat over the plane $z = 0$ into a semi-infinite medium and this similarity in form serves as a check of the method.

For the situation to be examined, q in Equation (5) is the result of abruptly altering incoming radiation, so that a balance of heat flow must be re-established at the surface. Fig. 3 is a rough sketch of a small portion of surface illustrating incoming radiation, $q_1$, a portion of which is absorbed, $q_2$, the surface reradiating energy as a function of its emissivity and temperature, $q_3 = \sigma\epsilon T^4$, and being supplied heat from the semi-infinite medium

$q_4$. For a change in conditions, since finite quantities of heat are absorbed or released by the medium materials in changing temperature, the time required for arrival at a new equilibrium state is governed by certain medium properties, including thermal conductivity, and specific heat. The reduced Equation (5) gives a means of estimating the effect of these properties. The value of q to be used in this equation is to be a net energy transfer at the surface, $q_2 - q_3$, which is a function of surface temperature ($q_3 - \sigma\epsilon T^4$). Fig. 4 plots the net heat interchange values for three cases of interest. These cases differ in the method of altering radiation on surface. To make use of the variable heat interchange associated with altering the incoming radiation, Equation (5) is set up in a difference form, realizing that, lacking an analytic solution including a variable heat function, the assumption will be made that Equation (5) can be applied at successive times with q taken to be a function of temperature. This gives,

$$\theta_n - \theta_{n-1} = \frac{2q_{n-1}}{\pi^{1/2}K} [\{\alpha n\Delta t\}^{1/2} - \{\alpha(n-1)\ \Delta t\}^{1/2}] \qquad (10)$$

$$\Delta\theta_n = \frac{2q_{n-1}}{\pi^{1/2}K} (\alpha\Delta t)^{1/2}[n^{1/2} - (n-1)^{1/2}], \qquad (11)$$

which relates the change in temperature, $\Delta\theta_n$, in a time step, $\Delta t$, to the properties of the medium, the net heat interchange existing at the beginning of the step, $q_{n-1}$, and the number of steps taken. By use of the data of Fig. 4 and the difference equation were obtained the curves of Figs. 5, 6, and 7 illustrating the temperature histories at the center of an area of altered radiation for the conditions noted. It should be possible in any of these cases to deduce thermal diffusivity with fair accuracy, perhaps within about 10%, having obtained radiometrically the temperature history. A more exact statement of precision available can be made after operating characteristics of the radiometers are know, but it is presumed that temperatures will be known to within ±5°C. The curves of Figs. 5, 6, and 7 give supporting evidence for the previous statement that the measurement of thermal diffusivities can be accomplished in values of $\alpha t \leq 1$. Going back to the original Carslaw and Jaeger solution for the point (0, 0, 0),

$$\theta = \frac{2q(\alpha t)^{1/2}}{K\,\pi^{1/2}}\left(1-e^{-a^2/4\alpha t}\right) + \frac{q\,a}{K}\ \mathrm{erfc}\,\frac{a}{2(\alpha t)^{1/2}}\,. \tag{4}$$

Note that for $e^{-\xi^2}$ may be substituted the series $\Sigma_{n=o}^{\infty}\frac{(-1)^n\xi^{2n}}{n!}$

performing this substitution gives,

$$\theta = \frac{2q}{\pi^{1/2}K}\left[(\alpha t)^{1/2}-(\alpha t)^{1/2}\left\{1-\frac{a^2}{4\alpha t}+\frac{a^4}{32\alpha^2t^2}-\frac{a^6}{384\alpha^3t^3}+\ldots\right\}\right] + \frac{q\,a}{K}\ \mathrm{erfc}\,\frac{a}{2(\alpha t)^{1/2}} \tag{12}$$

$$\theta = \frac{2q}{\pi^{1/2}K}\left(\frac{a^2}{4(\alpha t)^{1/2}}-\frac{a^4}{32(\alpha t)^{3/2}}+\frac{a^6}{384(\alpha t)^{5/2}}-\ldots\right) + \frac{q\,a}{K}\ \mathrm{erfc}\,\frac{a}{2(\alpha t)^{1/2}}\,, \tag{13}$$

thus, as $t \to \infty$ $\qquad \theta \to \frac{q\,a}{K}$ (14)

Accepting this as the steady state condition after long time periods, it is now possible to estimate the feasibility of determining the thermal conductivity of the surface materials, as distinguished from thermal diffusivity. In the equation, $\theta = \frac{q\,a}{K}$, the net heat interchange q, at the surface is given by $\sigma(T^4_{surface} - T^4_{shield})$ if the use of the shield is assumed. Using $T_{shield} = 300°K$, assuming the original surface temperature to be 400°K, the radius to be 10 cm., and computing the equilibrium condition by a trial and error procedure, it is found that the surface temperature falls within 7°C of the shield temperature at equilibrium for a conductivity $K = 10^{-4}$ calories/cm. sec. °C, and that for $K \leq 10^{-5}$ the surface temperature is within 1°C of the shield temperature. With the radiometer precision available it is not possible to discriminate conductivities by this method. This tendency for the low conductivity material to come to radiative equilibrium with an applied source holds also for the cases where a point source of infrared is used (e.g., lamps), therefore, it is felt that only a measurement of thermal diffusivity is practical on the surface.

## APPENDIX B

## DOWNHOLE DETERMINATION OF THERMAL DIFFUSIVITY

To estimate the thermal diffusivity from within a hole bored into the lunar body, a small blackbody radiator is attached to the bottom of the sonde. With a constant power input to this blackbody an increase in temperature with time will be noted, this temperature increase being a function of the conduction of heat away from the source along leads and supports, storage of heat in the sonde and blackbody materials, radiation of heat between blackbody surface and borehole wall, storage of heat within the lunar body, and conduction of heat away from the borehole through the lunar materials.

The design of the blackbody system is being done empirically, but as a start an investigation of shape factors was made for heat sources in insulating unconsolidated materials. As a means of prediction, use was made of the Carslaw and Jaeger[1] solution for the continuous spherical source within an infinite medium.

$$\theta = \frac{q}{8\pi\rho c\alpha r a}\left\{2\left(\frac{\alpha t}{\pi}\right)^{1/2}\left[e^{-\frac{(r-a)^2}{4\alpha t}} - e^{-\frac{(r+a)^2}{4\alpha t}}\right]\right.$$

$$\left. - |r-a| \operatorname{erfc}\frac{|r-a|}{2(\alpha t)^{1/2}} + (r+a)\operatorname{erfc}\frac{r+a}{2(\alpha t)^{1/2}}\right\}, \qquad (1)$$

with a constant rate of heating of q calories over the surface of a sphere of radius a starting at t = 0, the spherical source being in an infinite medium of density, $\rho$, specific heat c, and thermal

[1]Conduction of Heat in Solids, H. S. Carslaw and J. C. Jaeger, Second Edition, Oxford at the Clarendon Press, 1959, pp 263.

diffusivity $\alpha = \frac{K}{\rho c}$ (K = thermal conductivity), giving the temperature rise $\theta$ at distance r as a function of time, t. Evaluating this solution for r = a,

$$\theta = \frac{q}{8\pi\rho c\alpha a^2} [2(\frac{\alpha t}{\pi})^{1/2} [1-e^{\frac{-a^2}{\alpha t}}] + 2a \text{ erfc } \frac{a}{(\alpha t)^{1/2}}]. \qquad (2)$$

Rewriting,

$$\theta = \frac{q}{4\pi^{3/2}\rho c\alpha a} \{\frac{(\alpha t)^{1/2}}{a}\} [1-e^{-a^2/\alpha t} + \frac{a\pi^{1/2}}{(\alpha t)^{1/2}} \text{ erfc } \frac{a}{(\alpha t)^{1/2}}]. \qquad (3)$$

Setting $\xi = \frac{a}{(\alpha t)^{1/2}}$.

$$\theta = \frac{q}{4\pi^{3/2}\rho c\alpha a} \{\frac{1}{\xi}\} [1-e^{-\xi^2} + \pi^{1/2} \xi \text{ erfc } \xi]. \qquad (4)$$

The function $\frac{1}{\xi}(1-e^{-\xi^2} + \pi^{1/2} \xi \text{ erfc } \xi)$ is plotted in Fig. 8. Use of this equation allows prediction of heating curves for a spherical source in media of known properties. By use of the original equation it is proven easily that a surrounding medium giving a covering 25 cm. thick in every dimension is effectively an infinite medium for values of $\alpha t < 20$, and for values of $q \approx 0.25$ calories per second.

Three shapes were selected for study on the basis of sonde requirements for dimensions and configurations and were prepared in copper drilled out to give equal masses of metal. The heating element placed in each was a 2 watt, 100 ohm, carbon resistor "potted" in place with epoxy. Iron constantan thermocouple wires measure the temperature rise at selected points. Table I gives the data on each of the three shapes, Fig. 9 shows the configuration, and Figs. 10, 11, and 12 show comparison with the Carslaw and Jaeger prediction of the heating in media of different conductivities. For the two shapes which are not spherical the predicted temperature rise is computed from the Carslaw and Jaeger solution for a spherical source, taking a value for radius which is the radius of a sphere having equal surface area. In the extension of this study the effect of borehole and coupling to sonde will be simulated by working in evacuated glass tubing buried in media of differing conductivity. This, it should be emphasized, will allow a sufficiently good design prediction, but more exact work is required in the calibration.

## TABLE I

### SHAPE DATA FOR HEAT SOURCE INVESTIGATION

| Shape | Radius cm. | Length of Cylinder cm. | Slant Height cm. | Altitude cm. | Volume $cm^3$ | Surface Area $cm^2$ | Radius of a Sphere of Equal Surface (cm.) | Weight of Drilled Shape gms. |
|---|---|---|---|---|---|---|---|---|
| Sphere | 1.11 | | | | 5.73 | 15.5 | | 38.7 |
| Cylinder | 1.11 | 2.23 | | | 8.64 | 23.3 | 1.36 | 38.5 |
| Cylinder Cone | 1.12 | 1.26 | 2.23 | 1.92 | 7.36 | 20.4 | 1.27 | 38.6 |

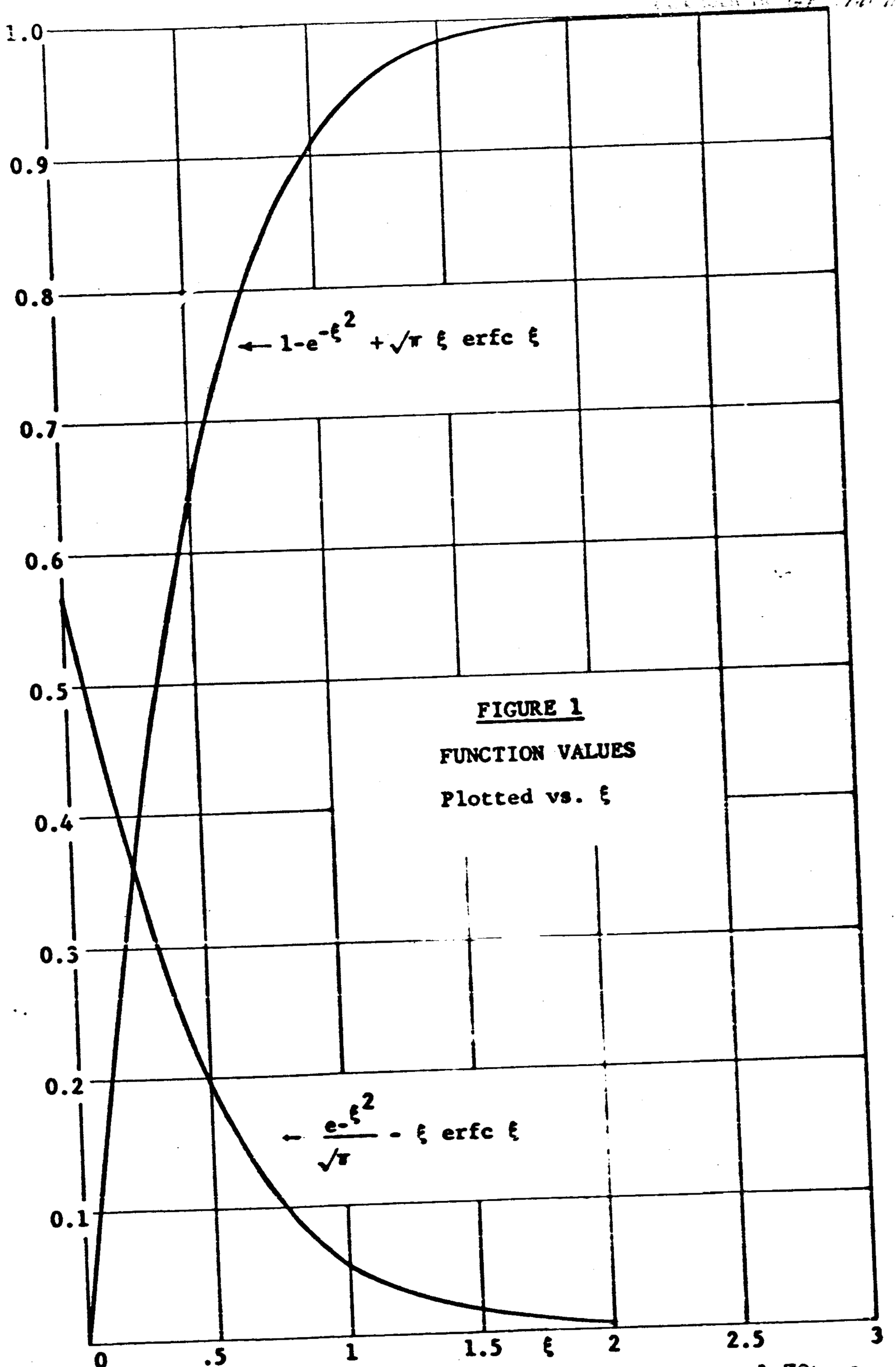


**FIGURE 1**

FUNCTION VALUES

Plotted vs. ξ

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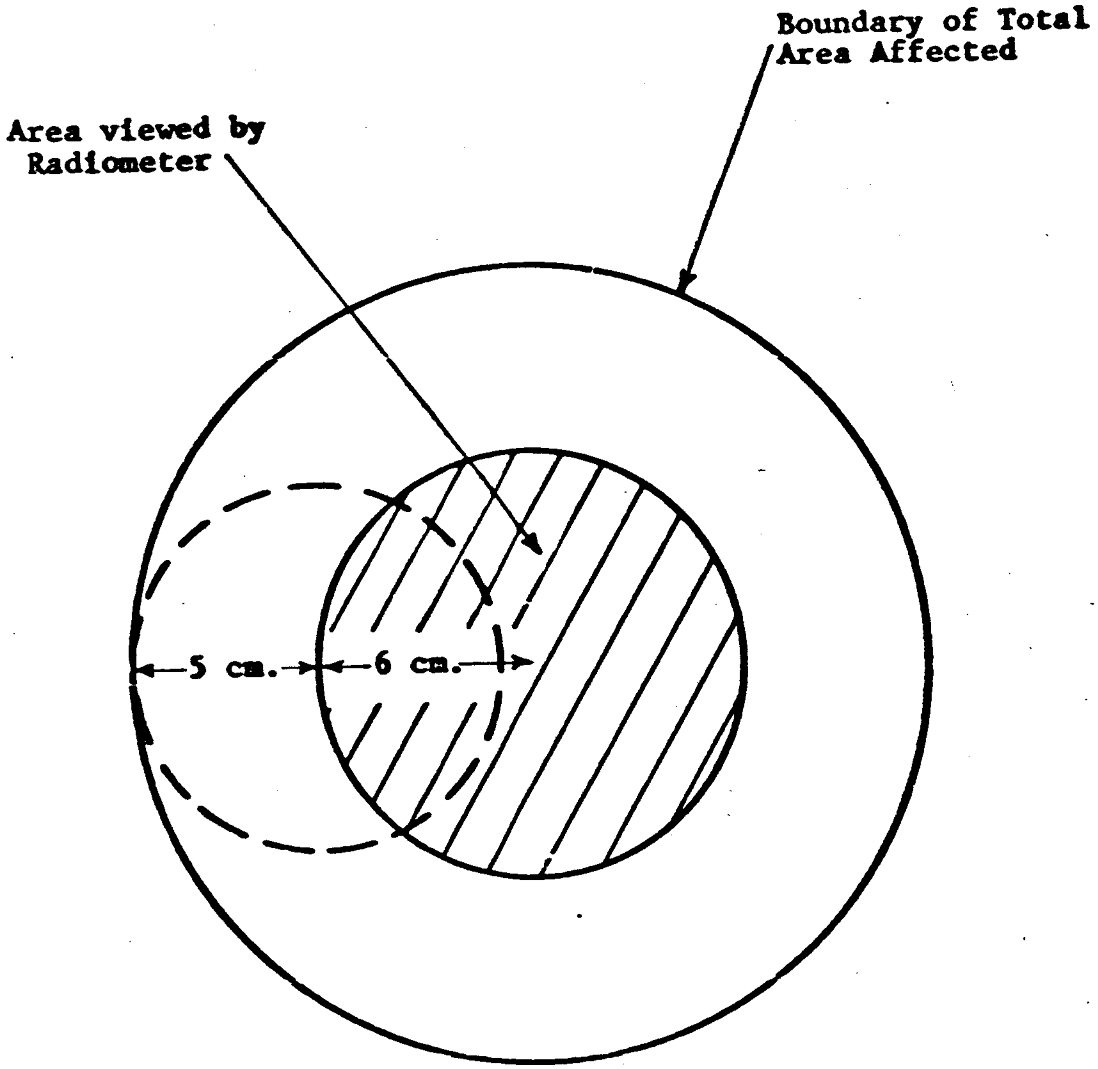


FIGURE 2

ILLUSTRATING AREA OF VIEW
AND AREA OF ALTERED RADIATION

1:794.28-F2

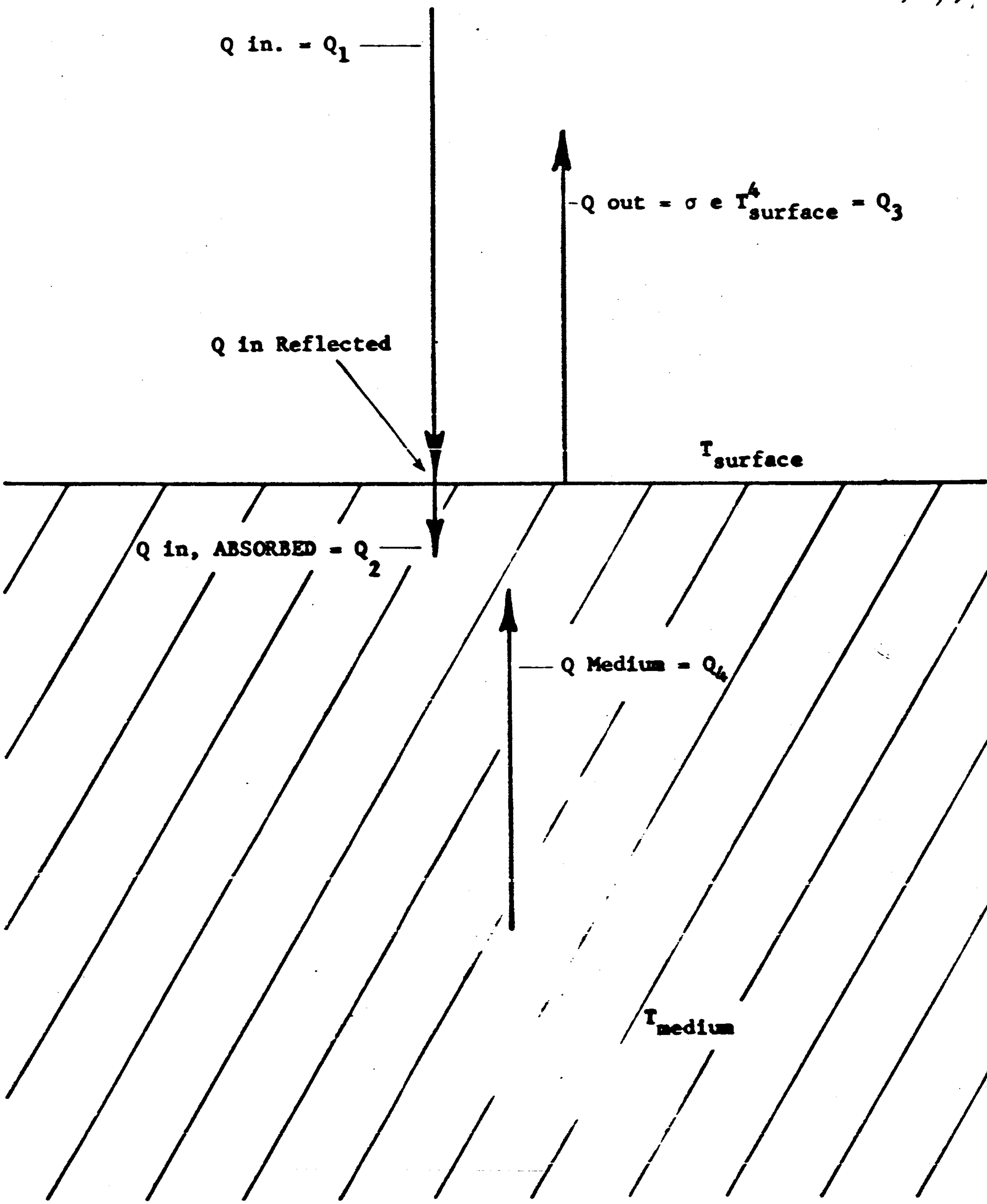


**FIGURE 3**

ENERGY INTERCHANGE AT SURFACE 1:794.28-F3

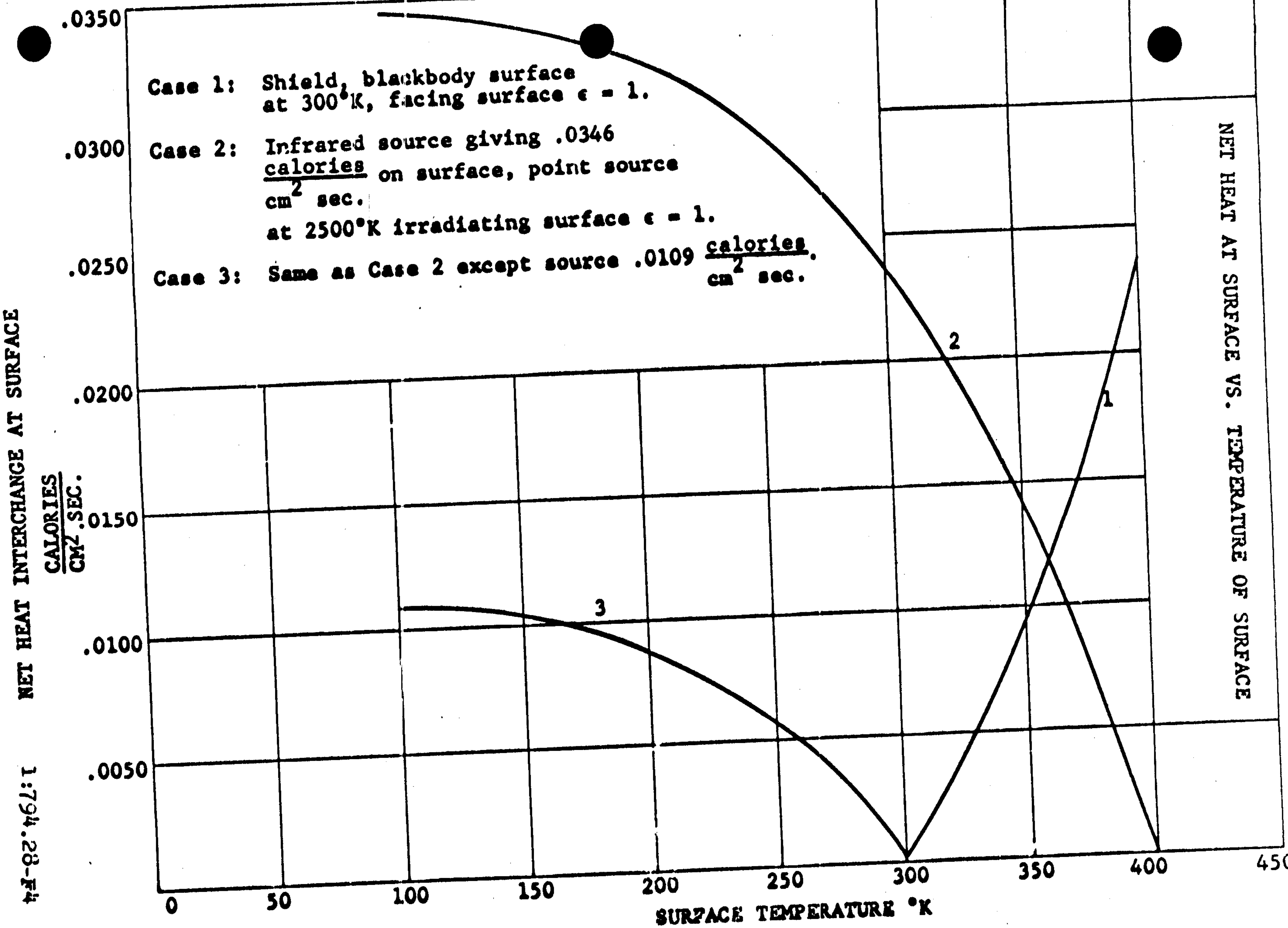


FIGURE 4

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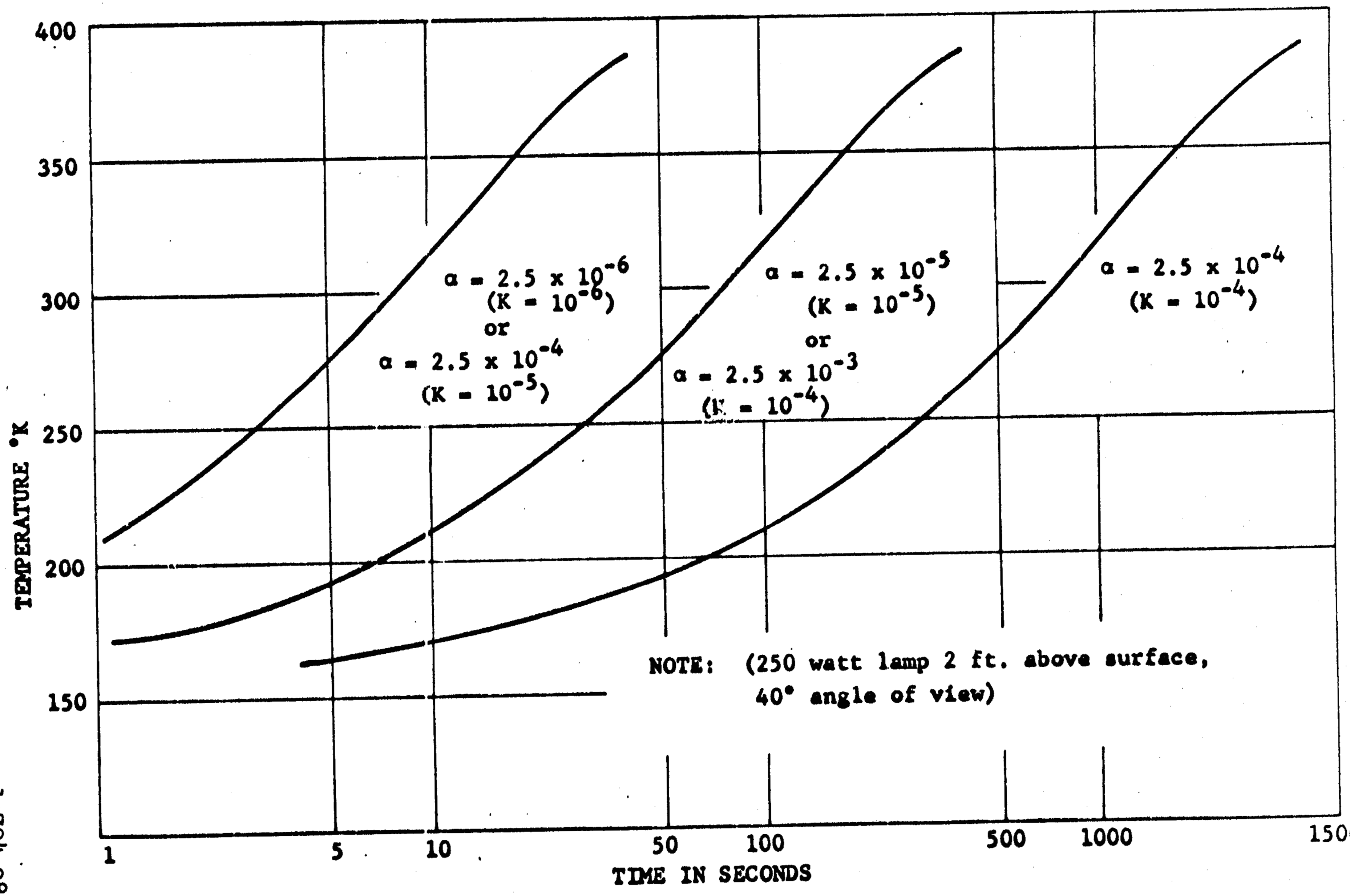



FIGURE 5

INFRARED SOURCE .0346 CALS/CM$^2$ SEC.
SURFACE ORIGINALLY AT 150°K

1:794.28-F5

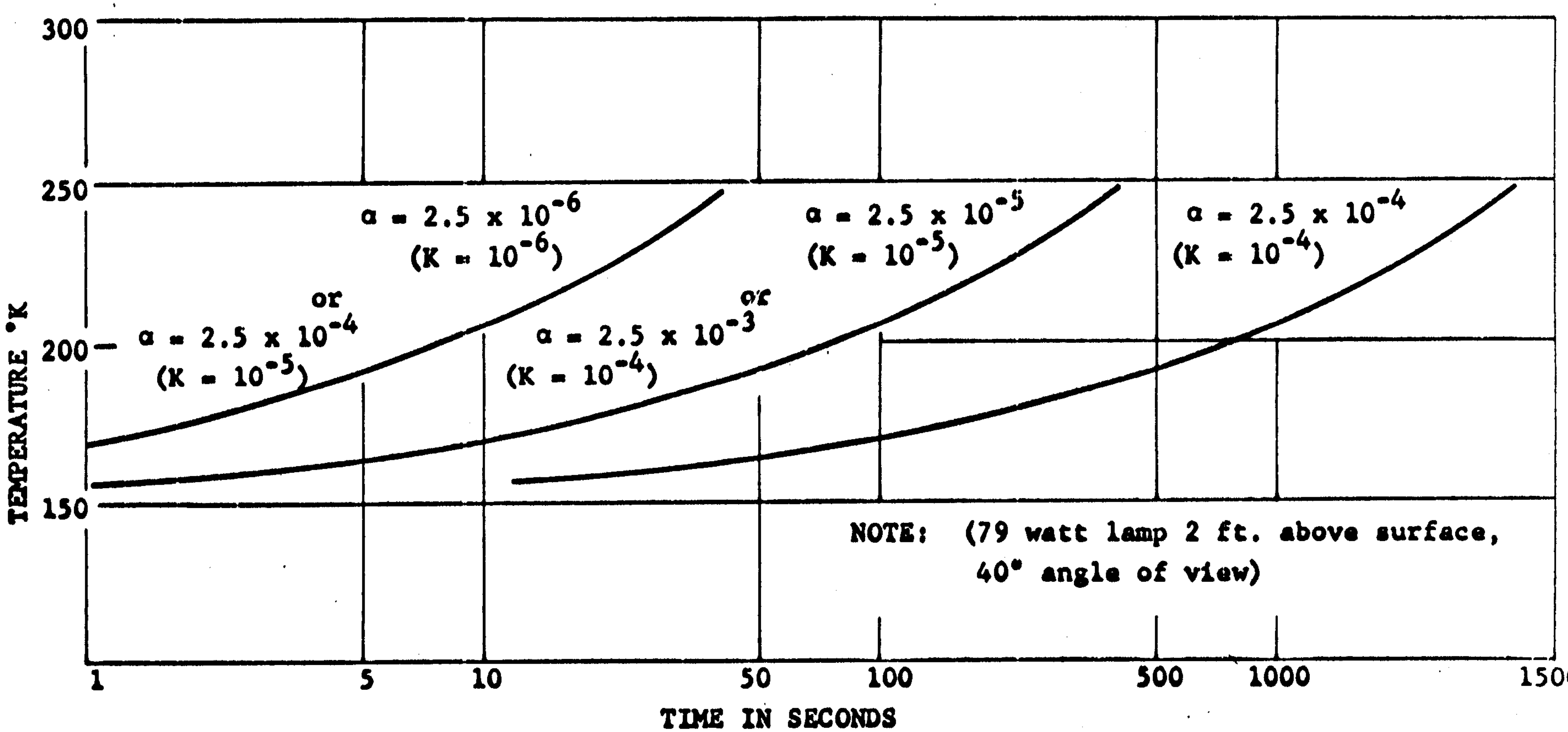



FIGURE 6

INFRARED SOURCE .0109 CALS/CM$^2$ SEC.
SURFACE ORIGINALLY AT 150°K

1:794.28-F6

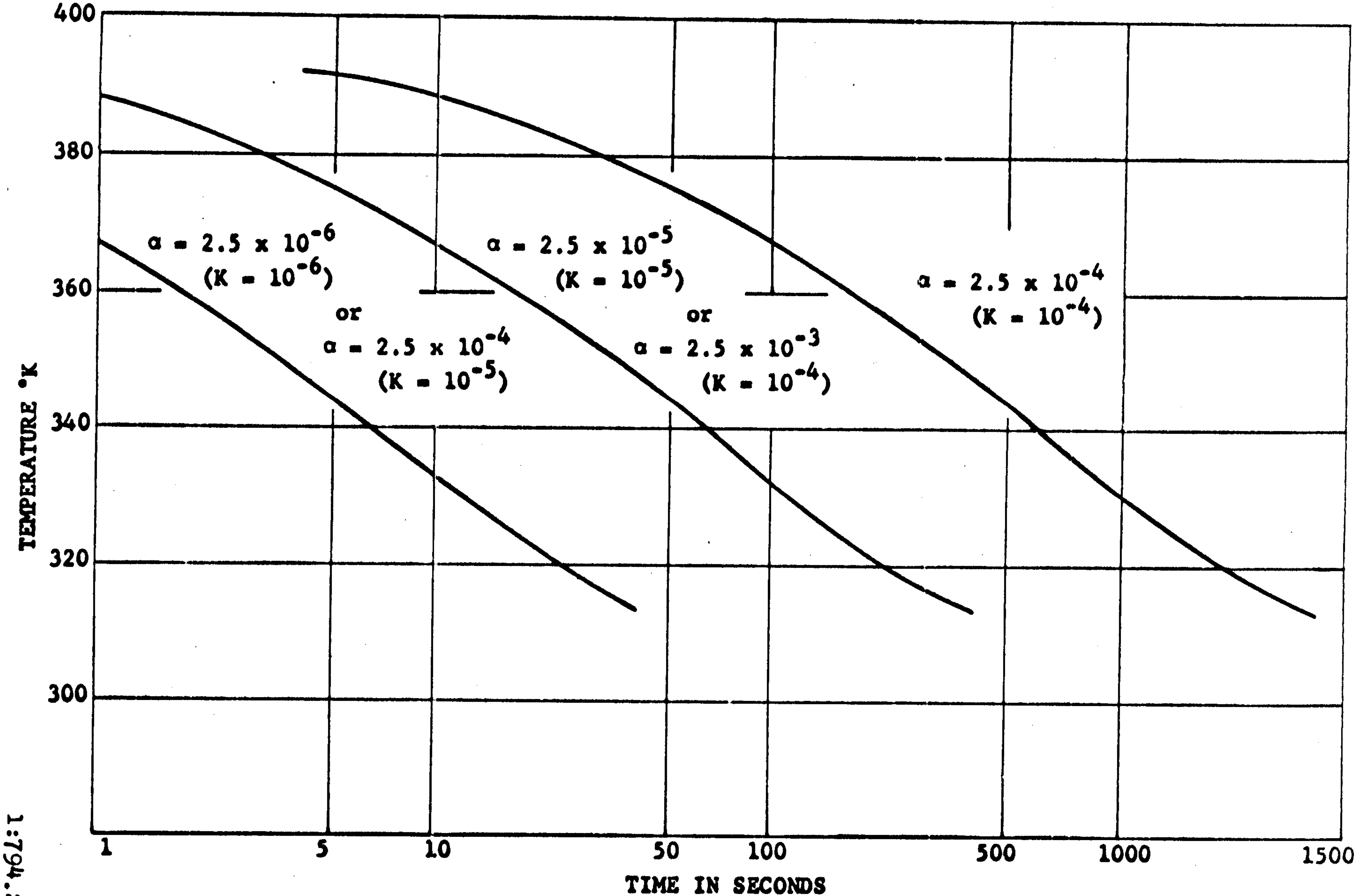



FIGURE 7

SHIELD AT 300°K, SURFACE ORIGINALLY
AT 400°K, WITH E = 1.0

1:794.28-F7

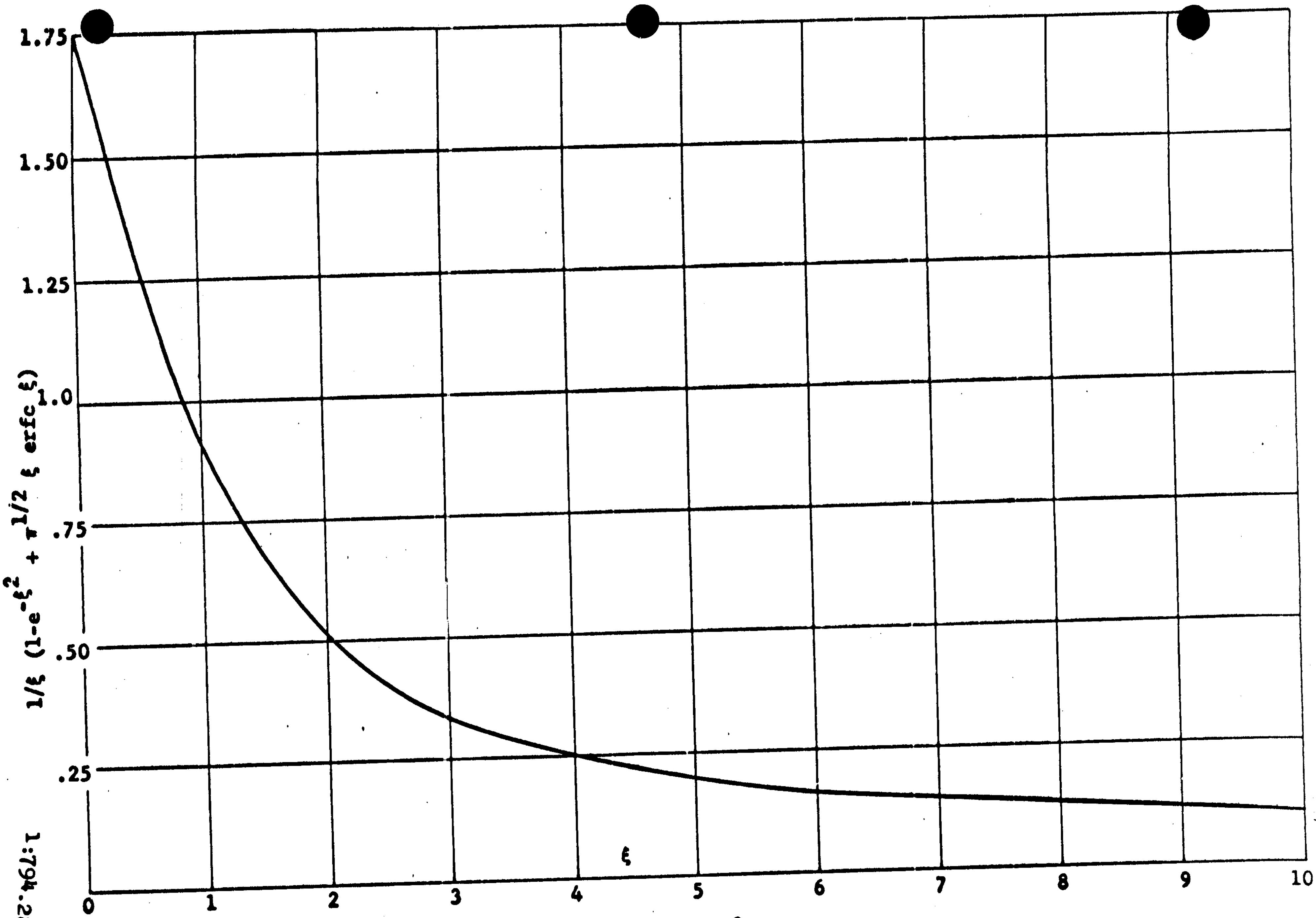


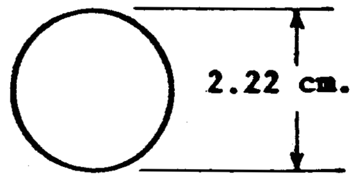

**FIGURE 8**

The function $1/\xi\ (1-e^{\xi^2} + \pi^{1/2}\ \xi \text{ erfc } \xi)$
plotted against $\xi$

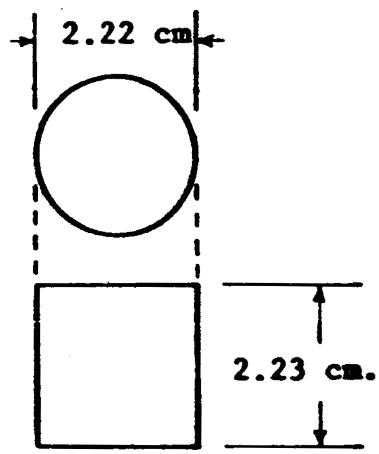
FIGURE 9

SHAPES SELECTED FOR HEAT SOURCE STUDIES

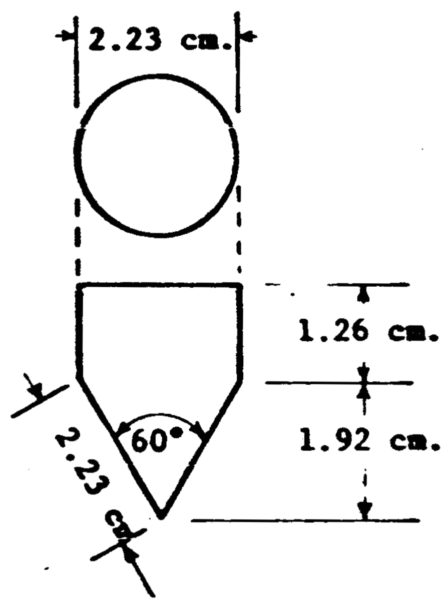
I. SPHERE




II. CYLINDER




III. CYLINDER - CONE




1:794.28-F9

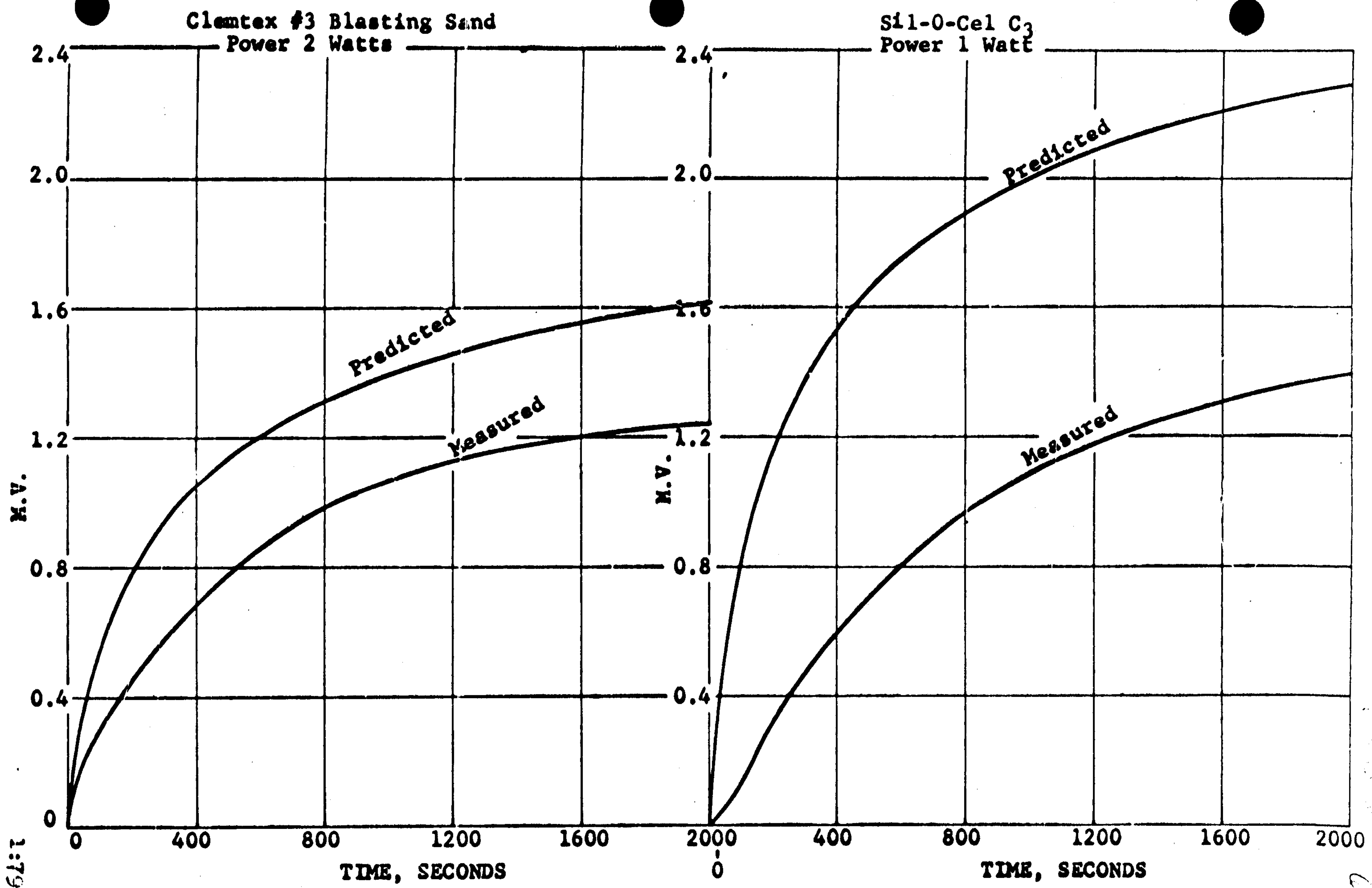



FIGURE 10

SPHERICAL SOURCE

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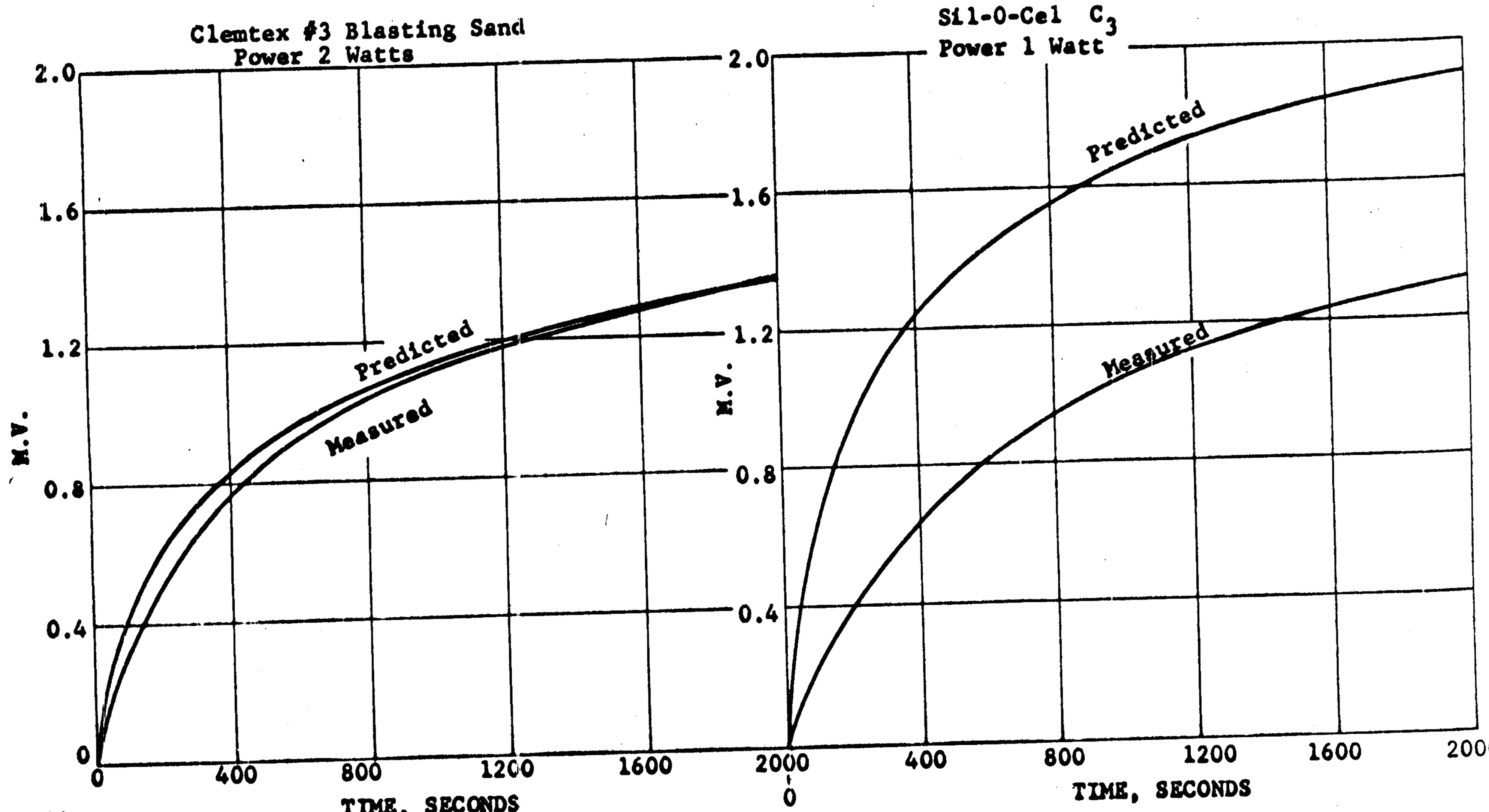


FIGURE 11

CYLINDER - CONE SOURCE

1:794.28-F11

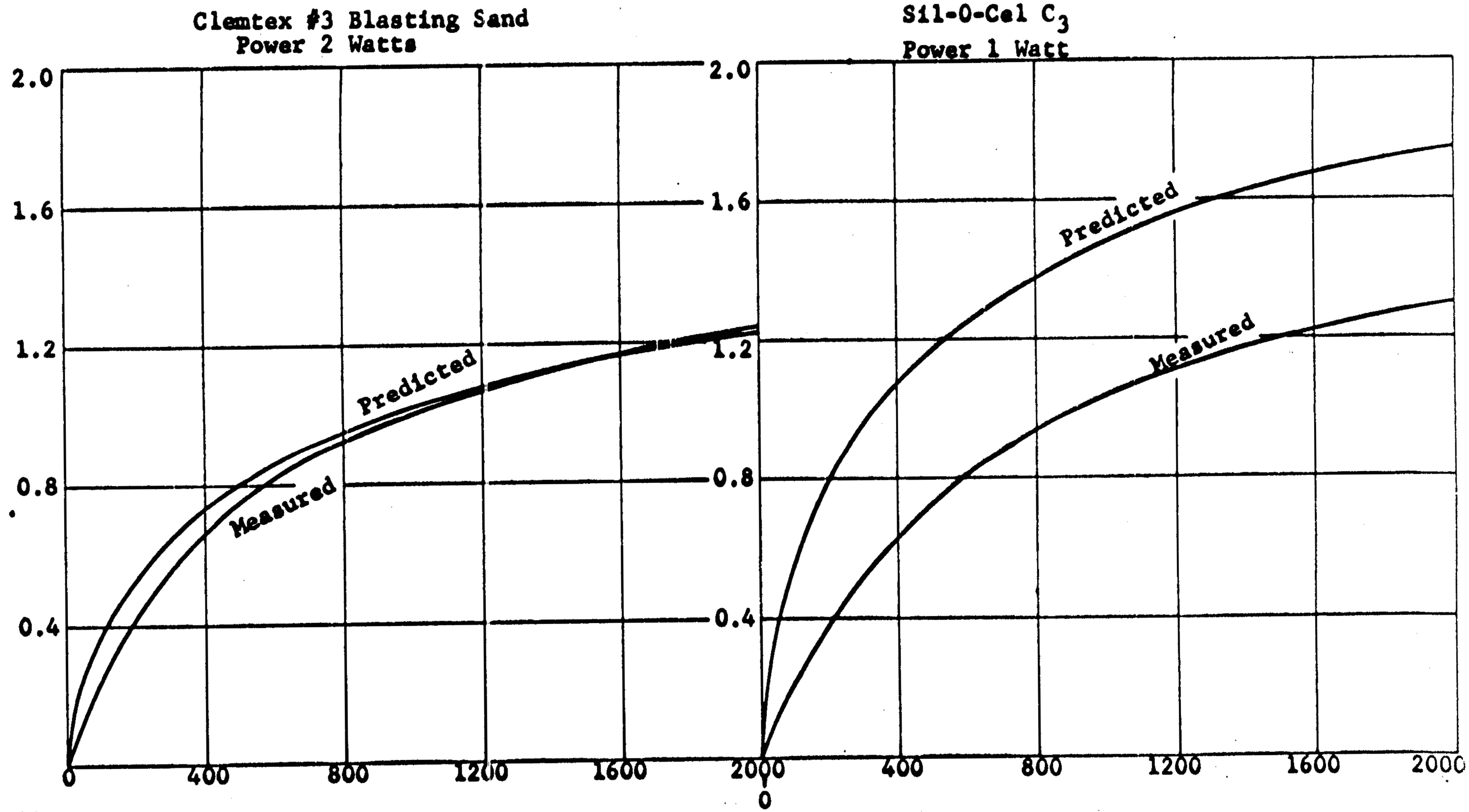


FIGURE 12

CYLINDER SOURCE

1:794.28-F12

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